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THESIS

ANALYTICAL TREATMENT OF
GRAVITY EFFECTS ON GAS LOADED
VARIABLE CONDUCTANCE HEAT PIPES

by

David Charles Kane

December 1980

Thesis Advisor:

M. Kelleher

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Analytical Treatment of
Gravity Effects on Gas Loaded
Variable Conductance Heat Pipes

by

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Submitted in partial fulfillment of the
requirements for the degree of

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ABSTRACT

The formulation of a one-dimensional analytical model representing steady state operation of a vertically oriented gas loaded variable conductance heat pipe is presented. The model includes not only the effects of binary mass diffusion and axial pipe-wall conduction, but also gravitational effects which exist at the vapor-gas interface region when the molecular weights of the working fluid and the noncondensable gas differ significantly. Analytical expressions for the transport processes (mass, momentum, and energy) along with equations of state result in a system of ten equations which describe steady state operation of the device. Combining these expressions yields a system of two simultaneous highly non-linear differential equations. These equations are non-dimensionalized and set up for numerical integration using a variable step fourth order Runge Kutta method as part of IBM's Continuous System Modeling Program (CSMP).

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LIST OF SYMBOLS

Ac	- cross-sectional area of vapor space
Ap	- axial cross-sectional area of wick-wall combination
C	- constant of integration
cs	- control surface
D	- binary mass diffusion coefficient
DY	- spatial derivative of Y
Dv	- diameter of vapor space
F	- external force
Fa	- saturation pressure as a function of temperature
g	- gravitational constant
G	- a dimensionless function of temperature
h	- convection heat transfer coefficient
h_{fg}	- heat of vaporization
k	- coefficient of thermal conductivity for wick-wall combination
K	- a dimensionless constant
L	- length of heat pipe condenser
\dot{m}	- condensation mass flux
M_b	- molecular weight of noncondensable gas
p	- pressure
Ps	- periphery of outside of pipe
Pv	- periphery of vapor space
q	- axial conduction heat flux
Q	- power level (heat rejection rate)

Q_c - rate of axial conduction in wick-wall combination
 R - universal gas constant
 S_a - working fluid saturation density as a function of temperature
 T - temperature (absolute)
 v - velocity
 x - axial position
 X_S - dimensionless length (independent variable)
 Y_1 - dimensionless temperature (dependent variable)
 Y_2 - $\ln \omega_a$ (dependent variable)
 Y_3 - spatial derivative of Y_1 ($Y_3 = DY_1$)
 Y_4 - spatial derivative of Y_2 ($Y_4 = DY_2$)
 V - volume

Greek symbols

μ - viscosity
 ρ - density
 τ - viscous shear force
 ω - mass fraction

subscripts

a - working fluid
 b - noncondensable gas
 e - evaporator
 ∞ - ambient conditions
 o - value at $x = 0$

I. INTRODUCTION

A. BACKGROUND

The heat pipe is a heat transfer device capable of transferring large quantities of heat under nearly isothermal operating conditions. This device is basically a liquid-vapor two phase system which transfers heat by evaporation and condensation of a working fluid at opposing ends of a tube. There are no moving parts in a heat pipe. To sustain operation the condensed working fluid is transferred from the condenser to the evaporator section of the device by capillary pumping action through a wick structure lining the heat pipe inner wall.

B. CONVENTIONAL HEAT PIPE PRINCIPLES

The conventional heat pipe in its simplest form consists of a hollow tube with wicking material lining its inner surface as shown in Figure 1. This device is evacuated, then filled with an appropriate amount of a working fluid.

In operation, heat at a given rate from a source is absorbed at one end of the pipe evaporating the fluid from the wick. The vapor then flows along the pipe to the condenser end where heat is rejected to a sink and the vapor is condensed into the wick. Circulation of the working fluid is maintained by a capillary head in the wick which must exactly balance pressure drops in the liquid and vapor, as well as any body

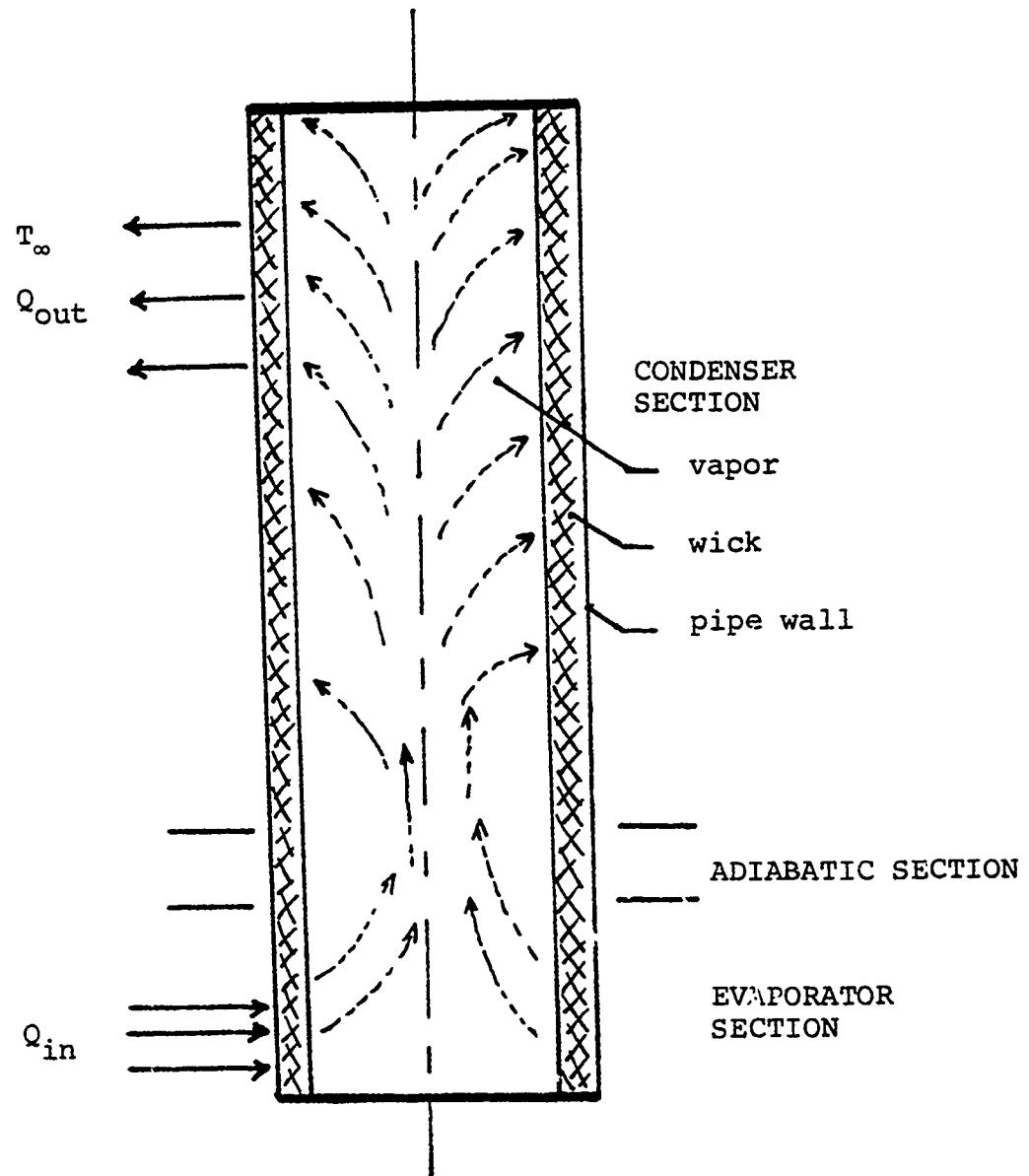


Figure 1. Conventional Heat Pipe

forces present. The heat load alone controls the saturation temperature and pressure of the working fluid, which become the operating temperature and pressure of the pipe.

Since the transfer of heat is effected by adding and removing the heat of vaporization at saturation conditions, the heat pipe is nearly isothermal along its length. However, a change in the heat load will automatically change the operating temperature (and pressure) of the pipe, given a constant sink temperature. This characteristic is undesirable for many applications where a constant source temperature is required.

C. VARIABLE CONDUCTANCE HEAT PIPE PRINCIPLES

1. General

If a noncondensable gas is introduced into a heat pipe along with the working fluid, the heat pipe becomes a passively controlled device which can operate at various heat loads while maintaining a more nearly constant source temperature.

During operation the working fluid behaves as described above; however, the noncondensable gas is entrained in the vapor flow and forms a gas plug at the condenser end of the pipe. This plug acts as a diffusion barrier to the flowing vapor so that the effective condenser length, and therefore condenser heat transfer area, is reduced. For a given heat load and noncondensable gas loading, the pipe will operate at a specific temperature. If the heat load is increased the

saturation temperature and partial pressure of the working fluid, as well as total pressure in the pipe, tend to increase. In response, the gas plug is compressed which increases the effective condenser heat transfer area, thus reducing the system temperature rise. A thorough presentation on the theory and design of variable conductance heat pipes has been provided by Marcus [1].

2. Flat Front Theory

Bienert [2] developed the earliest model used to examine the operating characteristics of a gas loaded heat pipe. This flat front theory assumed that the noncondensable gas was in the form of a plug with a sharply defined vapor-gas interface perpendicular to the axis of the pipe. This simple model neglected axial conduction in the heat pipe wall and the diffusion of vapor into the stagnant gas plug. This early model provided much insight into the operating characteristics of gas loaded heat pipes. However, experimental results [1] have shown the model to be inaccurate, primarily because it neglects axial conduction and diffusion.

3. Diffuse Front Theory

A more accurate model of the gas loaded heat pipe was provided by Edwards and Marcus [3]. The model, referred to as the 'diffuse front theory', includes effects of binary diffusion between the working fluid and the noncondensable gas as well as axial conduction in the pipe wall. Marcus reported [1] good correlation between experimental results and a computer program based upon the diffuse front theory.

4. Gravitational Effects

Neither the flat front theory nor the diffuse front theory included body force effects on the vapor-gas interface region. This simplification was probably made because much of the early work with heat pipes was related to space program applications.

Kelleher and Batts [4] reported convincing experimental results indicating that gravity effects can significantly distort the axial temperature profile when the working fluid and noncondensable gas are of significantly different molecular weights.

D. OBJECTIVE

The objective of this research was to develop a one-dimensional analytical model describing steady state operation of a gas loaded variable conductance heat pipe. The model was to include not only the effects of axial conduction and binary mass diffusion, but also gravitational effects on the vapor-gas interface region. An additional purpose of this effort was to transform the analytical equations into a set of non-dimensionalized simultaneous differential equations which could readily be solved numerically using IBM's Continuous System Modeling Program III [5].

II. FORMULATION OF ANALYTIC EQUATIONS

In formulating the analytic equations describing steady state heat pipe operation the differential control volume in Figure 2 was developed to represent a section of the heat pipe condenser. As implied in Figure 2, only vertically

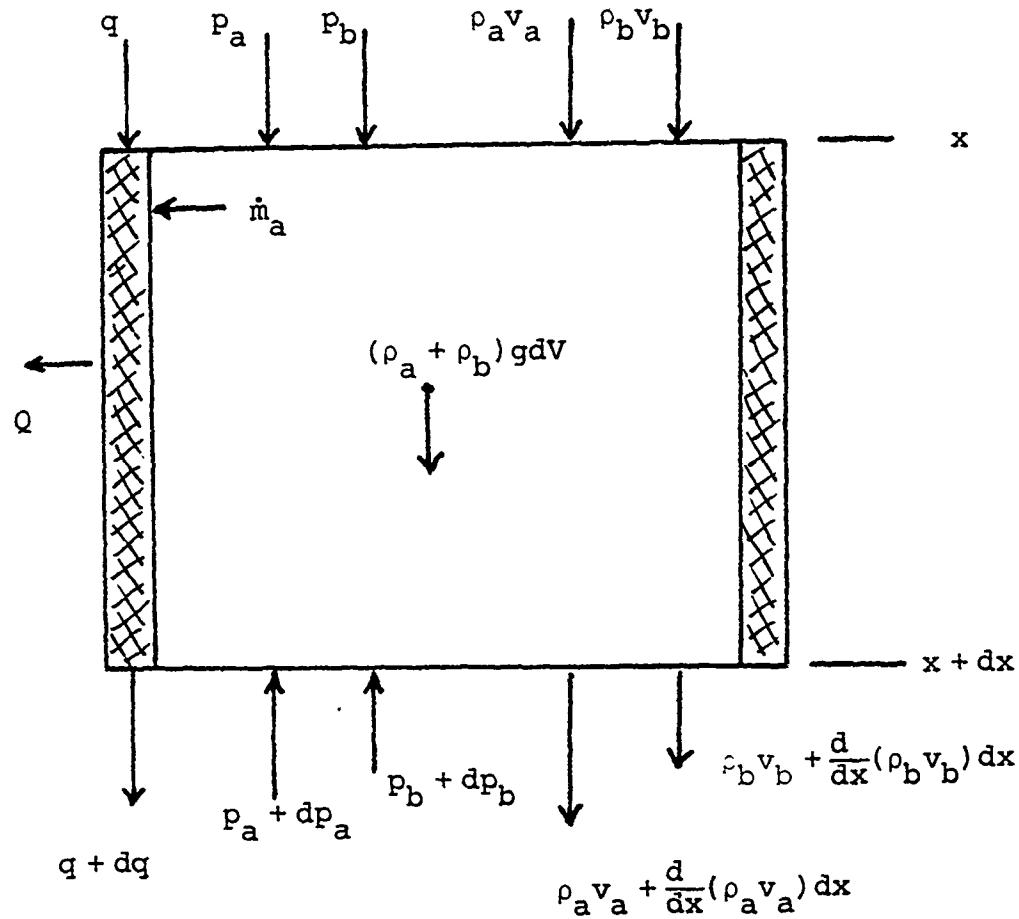


Figure 2. Differential Volume Element

oriented heat pipes are considered so that the transport processes can be treated as one dimensional.

A. SIMPLIFYING ASSUMPTIONS

The following assumptions were made in formulating an analytical model using the various transport processes:

1. The heat, mass, and momentum transport processes were assumed to be one-dimensional in the axial direction within the vapor space. This assumption restricted this analytical model to the specific case of a vertically oriented heat pipe. In any other orientation, gravitational effects should be expected to cause highly three dimensional spatial variations when the molecular weights of the working fluid and noncondensable gas differ substantially. This effect was demonstrated experimentally by Kelleher and Batts [4].
2. The working fluid and the noncondensable gas were treated as thermodynamically independent substances.
3. Steady state conditions were assumed.
4. The working fluid was considered to be at saturation conditions at all times.
5. Thermal resistance of the wick-wall structure, in the radial direction, was considered to be negligible compared to the thermal resistance outside of the pipe.
6. The noncondensable gas was treated as a perfect gas.

B. CONSERVATION OF MASS

1. Species 'a' (working fluid)

Referring to the differential volume element in Figure 2 and considering the system on a unit time basis,

$$\text{mass of 'a' in} = \text{mass of 'a' out}$$

or,

$$\rho_a v_a A c = [\rho_a v_a + \frac{d}{dx}(\rho_a v_a) dx] A c + \dot{m}_a P v dx \quad (\text{II.1})$$

which reduces to

$$0 = \dot{m}_a P v + A c \frac{d}{dx}(\rho_a v_a) \quad (\text{II.2})$$

Fick's first law of binary diffusion, as developed in reference 6, was used to relate the mass flux of working fluid ($\rho_a v_a$) to the mass flux of the mixture (ρv). For the one-dimensional case the scalar form of Fick's law can be written,

$$\rho_a v_a - \omega_a (\rho v) = -\rho D_{ab} \frac{d\omega_a}{dx} \quad (\text{II.3})$$

where the mass fraction of species 'a' is defined as,

$$\omega_a = \rho_a / \rho \quad (\text{II.4})$$

and the density of the mixture is given by

$$\rho = \rho_a + \rho_b \quad (\text{II.5})$$

since species 'a' and species 'b' occupy the same volume as independent substances.

Then equation II.2 becomes,

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}(\omega_a \rho v) - \frac{d}{dx}(\rho D_{ab} \frac{d\omega_a}{dx}) = 0 \quad (\text{II.6})$$

which represents the conservation of mass for species 'a'.

2. Species 'b' (noncondensable gas)

A similar equation was written for species 'b' noting that since species 'b' is a noncondensable gas, $\dot{m}_b \equiv 0$.

Then,

$$\frac{d}{dx}(\omega_b \rho v) - \frac{d}{dx}(\rho D_{ba} \frac{d\omega_b}{dx}) = 0 \quad (\text{II.7})$$

where the mass fraction of species 'b' is defined as,

$$\omega_b = \rho_b / \rho \quad (\text{II.8})$$

3. Mass Continuity of the Mixture

Adding the equations for species 'a' and species 'b' (III.6 and III.7) gave the continuity equation for the mixture,

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}[(\rho v(\omega_a + \omega_b))] - \frac{d}{dx}[\rho D(\frac{d\omega_a}{dx} + \frac{d\omega_b}{dx})] = 0 \quad (\text{II.9})$$

where $D = D_{ab} = D_{ba}$ is used for simplicity of notation.

Recalling equations II.4, II.5, and II.8,

$$\omega_a + \omega_b = 1$$

which reduced equation II.9 for mass continuity of the mixture to,

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}(\rho v) = 0 \quad (\text{II.10})$$

C. CONSERVATION OF MOMENTUM

1. Mixture of Species 'a' and Species 'b'

Conservation of linear momentum for the mixture, with steady state conditions, was written for the vapor space differential element of Figure 3 as,

$$\vec{\sum F} = \int_{cs} \vec{v} (\rho \vec{v} \cdot d\vec{A})$$

where, $d\vec{A}$ is the outward directed surface area element and, \vec{F} is the sum of all external forces acting on the fluid in the control volume. For the one-dimensional case $\vec{F} = \vec{F}_x$. The sum of the body forces and surface forces was seen from Figure 3 to be,

$$\vec{\sum F}_x = \rho g Ac dx - \tau Fv dx - dp Ac \quad (\text{II.11})$$

and the surface integral, representing the net efflux of linear momentum in the axial direction from the control

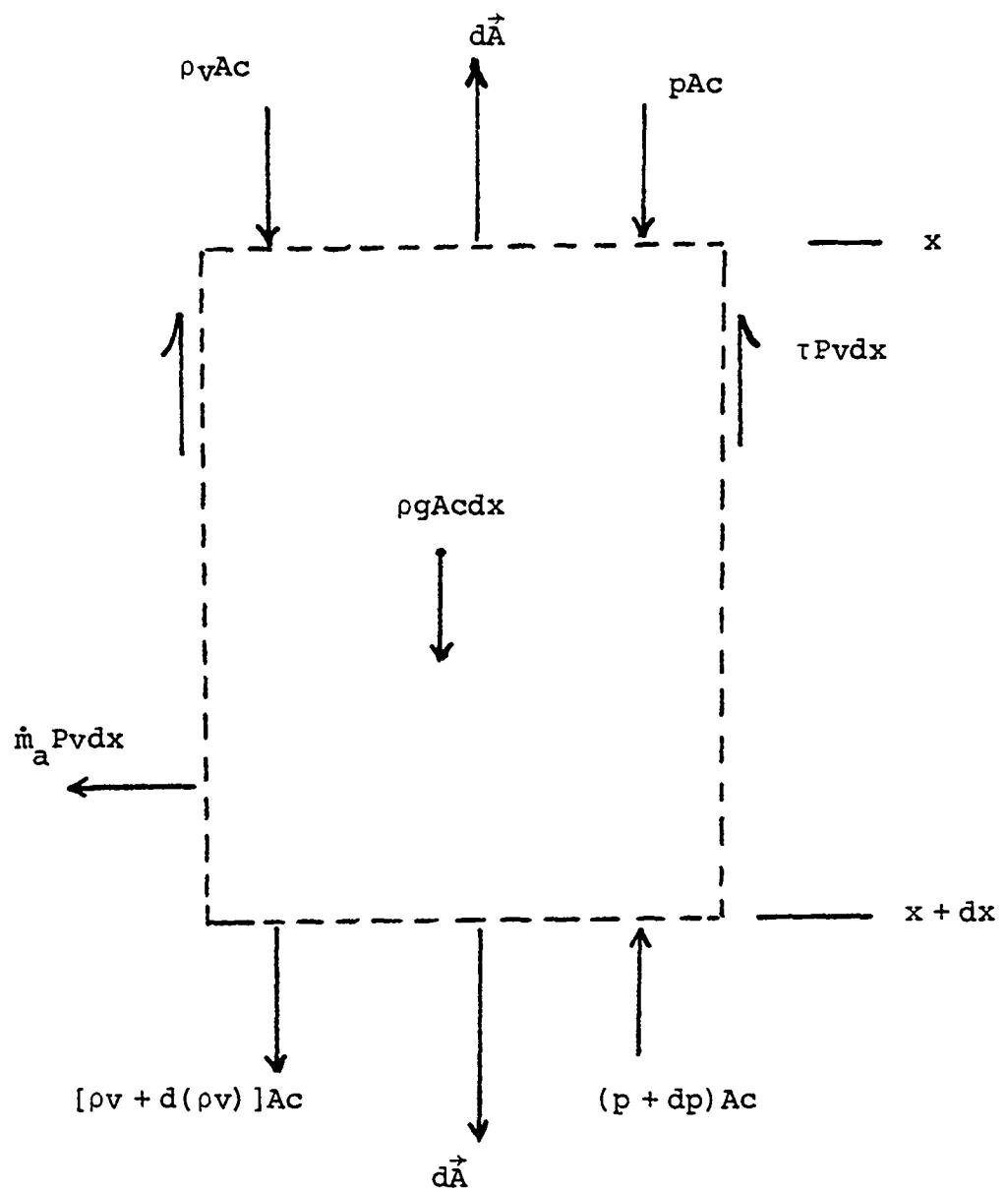


Figure 3. Vapor Space Differential Element

volume, was expanded to give:

$$\int_{cs, x} = - \rho v^2 A_c + (v + dv)(\rho v + d(\rho v))A_c + v_a \dot{m}_a P v dx \\ = A_c \rho v dv + A_c v d(\rho v) + v_a \dot{m}_a P v dx \quad (II.12)$$

equating equations II.11 and II.12, and rearranging yielded,

$$\rho g - \tau \frac{Pv}{A_c} - \frac{dp}{dx} = \rho v \frac{dv}{dx} + v \frac{d(\rho v)}{dx} + v_a \dot{m}_a \frac{Pv}{A_c} \quad (II.13)$$

For steady state conditions the noncondensable gas remains stationary with respect to the coordinates fixed to the heat pipe. Therefore the net flux of species 'b' is identically zero (i.e., $\rho_b v_b = 0$ since $v_b \equiv 0$). Noting that $\rho v = \rho_a v_a + \rho_b v_b$, and $v_b \equiv 0$,

$$\rho v = \rho_a v_a$$

or

$$v_a = \frac{\rho}{\rho_a} v = \frac{v}{\omega_a}$$

Replacing v_a in equation II.13 and rearranging resulted in the momentum equation for the mixture,

$$\frac{dp}{dx} = \rho g - \frac{d}{dx}(\rho v^2) - \frac{Pv}{A_c} v \frac{\dot{m}_a}{\omega_a} - \frac{Pv}{A_c} \tau \quad (II.14)$$

where the viscous shear force τ is some function of velocity.

For example, $\tau = \frac{8\mu}{Dv} v$, if Poiseuille flow is assumed.

2. Species 'b'

Since the velocity of species 'b' is zero, the conservation of momentum equation for species 'b' could be written directly by inspection as

$$\frac{dp_b}{dx} = \rho_b g \quad (\text{II.15})$$

A separate momentum equation for species 'a', similar in form to equation II.14 could be written. However the additional equation would not be useful since it would not be linearly independent of the two momentum equations already presented.

D. ENERGY BALANCE FOR PIPE WALL

In developing a heat pipe energy balance the wick and pipe wall were considered together as a composite. Recall that thermal resistance of the wick-wall structure, in the radial direction, was assumed to be negligible. Figure 4 then represents an appropriate differential element upon which to develop the energy balance.

By the conservation of energy (referring to Figure 4),

$$Q_{in} = Q_{out}$$

then,

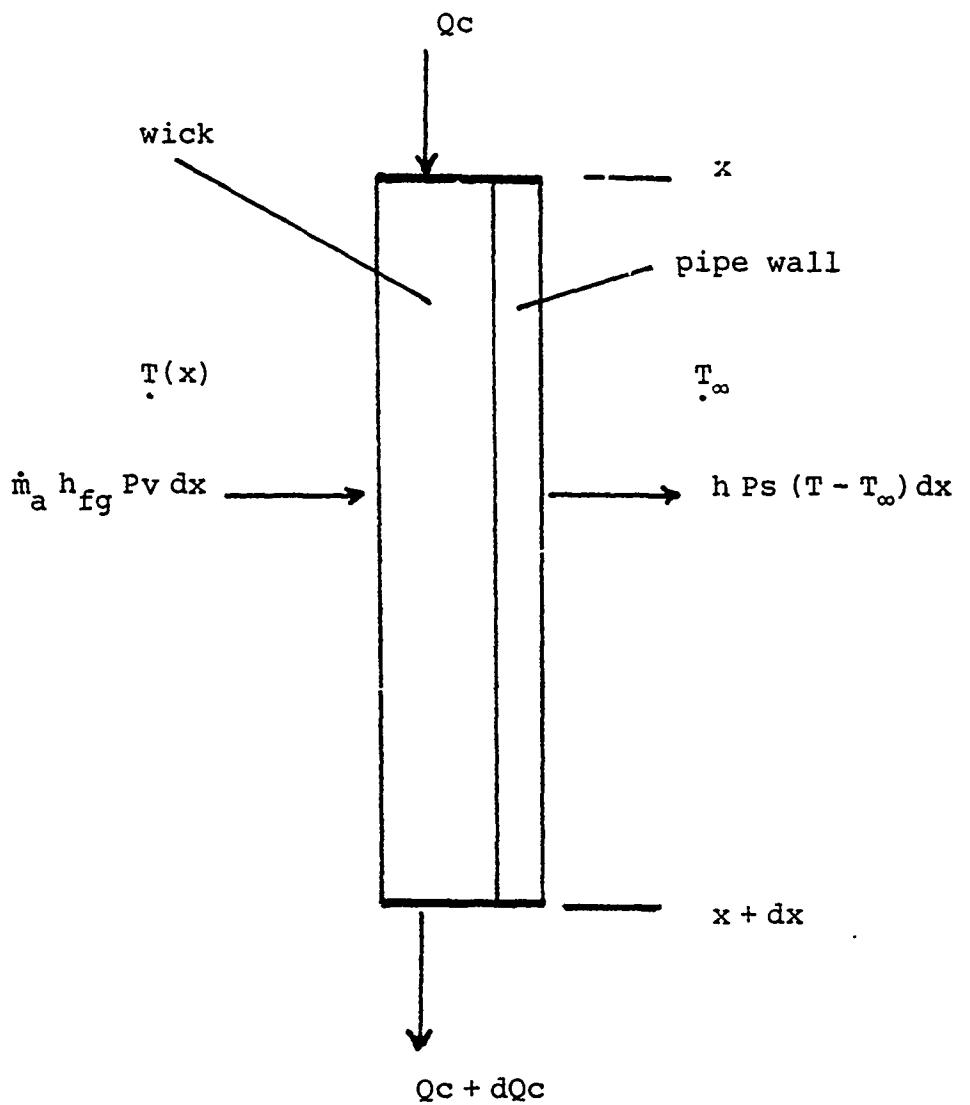


Figure 4. Differential Element for Energy Balance

$$Q_c + \dot{m}_a h_{fg} P_v dx = Q_c + dQ_c + h P_s (T - T_\infty) dx \quad (II.16)$$

but, by Fourier's law of conduction,

$$Q_c = - k A_p \frac{dT}{dx} \quad (II.17)$$

Incorporating equation II.17 into equation II.16 and rearranging yields the energy balance equation,

$$\frac{d^2T}{dx^2} - \frac{h P_s}{k A_p} (T - T_\infty) + \frac{h_{fg} P_v}{k A_p} \dot{m}_a = 0 \quad (II.18)$$

where,

- (a) the coefficient of thermal conductivity (k) must be selected to represent the composite wick-pipe wall structure, and
- (b) the heat transfer coefficient, h , is some function of temperature. The correlation selected for $h(T)$ depends upon the external cooling conditions.

E. EQUATIONS OF STATE

1. Species 'a'

Since saturation conditions were assumed to exist in the working fluid at all times, tabulated saturation properties of the working fluid can be used to generate functions for partial pressure, partial density and heat of

vaporization of the working fluid as functions of local temperature. Then,

$$p_a = F_a(T) \quad (\text{II.19})$$

$$\rho_a = S_a(T) \quad (\text{II.20})$$

The functions $F_a(T)$, $S_a(T)$, as well as $h_{fg}(T)$ represent actual tabulated saturation properties of the working fluid which can be readily used in a computerized interpolation scheme. Alternatively $F_a(T)$, $S_a(T)$, and $h_{fg}(T)$ could be approximated by polynomials or other analytic functions selected to fit the working fluid saturation curves.

2. Species 'b'

As a perfect gas,

$$p_b = \rho_b \frac{R}{M_b} T \quad (\text{II.21})$$

3. Mixture Properties

For independent substances,

$$p = p_a + p_b \quad (\text{II.22})$$

and,

$$\rho = \rho_a + \rho_b \quad (\text{II.23})$$

Although equation II.22 is valid based upon Dalton's law of partial pressures, it is included here for completeness only since it is not linearly independent of equations II.14 and II.15 developed from momentum considerations.

F. SUMMARY OF EQUATIONS

The following set of ten independent equations in ten unknowns describes steady state operation of a vertically oriented gas loaded heat pipe.

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}(\omega_a \rho v) - \frac{d}{dx}(\rho D \frac{d\omega}{dx}) = 0 \quad (\text{II.6})$$

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}(\rho v) = 0 \quad (\text{II.10})$$

$$\frac{dp}{dx} = \rho g - \frac{d}{dx}(\rho v^2) - \frac{Pv}{Ac} v \frac{\dot{m}_a}{\omega_a} - \frac{Pv}{Ac} \tau \quad (\text{II.14})$$

$$\frac{dp_b}{dx} = \rho_b g \quad (\text{II.15})$$

$$\frac{d^2T}{dx^2} - \frac{Ps}{k Ap} h (T - T_\infty) + \frac{h_{fg}}{k} \frac{Pv}{Ap} \dot{m}_a = 0 \quad (\text{II.18})$$

$$p_b = \rho_b \frac{R}{M_b} T \quad (\text{II.21})$$

$$\omega_a = \rho_a / \rho \quad (\text{II.4})$$

$$\rho = \rho_a + \rho_b \quad (\text{II.23})$$

$$p_a = F_a(T) \quad (\text{II.19})$$

$$\rho_a = S_a(T) \quad (\text{II.20})$$

The ten dependent variables (\dot{m}_a , v , ρ , p_a , ρ_b , p , p_a , p_b , ω_a , T) are all functions of axial position.

III. TRANSFORMATION TO COMPUTER MODEL

A. ELIMINATION OF VARIABLES

The ultimate goal was to obtain two nondimensionalized differential equations with nondimensional temperature and mass fraction of species 'a' as the sole dependent variables. By reducing the number of equations and dependent variables analytically the number of computations required for each numerical integration step is significantly reduced. This approach was taken to enhance the numerical accuracy and stability of computer results for the model. With this objective in mind, the set of ten equations in ten unknowns summarized in Section II.F was reduced to two equations in two unknowns by the following steps.

1. Total pressure (p) appeared in equation II.14 only and therefore could not be eliminated as a dependent variable by back substitution. Since total pressure as a function of axial position was not of particular interest, p was eliminated by assuming that the axial pressure gradient of total pressure was negligible ($dp/dx = 0$). Since the axial pressure gradient of total pressure is primarily created by the hydrostatic head of the vapor-gas mixture, this common assumption is reasonable. This assumption means that total pressure is assumed to remain essentially constant throughout the vapor-gas space of the pipe for a given operating power, gas load, operating temperature and sink temperature.

It is significant to note that although the axial gradient of total pressure is negligible, axial variations in the partial pressures of both working fluid vapor and noncondensable gas are significant. For example, using methanol as the working fluid at typical operating conditions results in an axial partial pressure variation of several atmospheres.

This assumption eliminated equation II.14 and the variable p .

2. The following simple substitutions were then sequentially made in the remaining equations.

a. Equation II.20 was used to eliminate ρ_a .

$$\rho_a = S_a(T)$$

b. Equation II.19 was used to eliminate p_a .

$$p_a = F_a(T)$$

c. Equation II.21 was used to eliminate p_b .

$$p_b = \rho_b \frac{R}{M_b} T$$

d. Equation II.23 was used to eliminate ρ_b .

$$\rho_b = \rho - \rho_a$$

e. Equation II.4 was used to eliminate ρ .

$$\rho = \frac{\rho_a}{\omega_a} = \frac{S_a(T)}{\omega_a}$$

3. The surviving four equations in four unknowns (\dot{m}_a , v, T, ω_a) were:

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}(Sa v) - \frac{d}{dx}\left(\frac{Sa D}{\omega_a} \frac{d\omega_a}{dx}\right) = 0 \quad (\text{III.1})$$

$$\dot{m}_a \frac{Pv}{Ac} + \frac{d}{dx}\left(\frac{Sa v}{\omega_a}\right) = 0 \quad (\text{III.2})$$

$$\frac{d}{dx}[Sa\left(\frac{1}{\omega_a} - 1\right)\frac{RT}{M_b}] = Sa\left(\frac{1}{\omega_a} - 1\right)g \quad (\text{III.3})$$

$$\frac{d^2T}{dx^2} - \frac{Ps h}{k Ap}(T - T_\infty) + \frac{h_{fg}}{k Ap} \dot{m}_a = 0 \quad (\text{III.4})$$

4. Equation III.2 was then used to eliminate \dot{m}_a in equations III.1 and III.4.

$$\dot{m}_a = -\frac{Ac}{Pv} \frac{d}{dx}\left(\frac{Sa v}{\omega_a}\right)$$

Equation III.1 then became:

$$-\frac{d}{dx}\left(\frac{Sa v}{\omega_a}\right) + \frac{d}{dx}(Sa v) - \frac{d}{dx}\left(\frac{Sa D}{\omega_a} \frac{d\omega_a}{dx}\right) = 0$$

which was integrated directly to yield,

$$Sa v(1 - \frac{1}{\omega_a}) - \frac{Sa D}{\omega_a} \frac{d\omega_a}{dx} = C$$

where C is a constant of integration. However, at $x = 0$

$$v = 0 \quad \text{and} \quad Sa/\omega_a = \rho_0$$

then

$$C = - D_0 \rho_0 \left. \frac{d\omega_a}{dx} \right|_0 \quad (\text{III.5})$$

where $x = 0$ was assumed to be at the condenser-reservoir end of the pipe. However, $\left. \frac{d\omega_a}{dx} \right|_0 = 0$ is a boundary condition as will be discussed later.

$$\text{Therefore } C = 0$$

The remaining system of three equations was then,

$$Sa v \left(1 - \frac{1}{\omega_a} \right) - \frac{Sa D}{\omega_a} \frac{d\omega_a}{dx} = 0 \quad (\text{III.6})$$

$$\frac{d}{dx} [Sa \left(\frac{1}{\omega_a} - 1 \right) \frac{R}{M_b} T] = Sa \left(\frac{1}{\omega_a} - 1 \right) g \quad (\text{III.3})$$

$$\frac{d^2 T}{dx^2} - \frac{Ps h}{k Ap} (T - T_\infty) - \frac{h_f g A_c}{k Ap} \frac{d}{dx} \left(\frac{Sa}{\omega_a} \right) = 0 \quad (\text{III.7})$$

5. Equation III.6 was finally solved for velocity and used to eliminate v in equation III.7 to give a system of two equations in T and ω_a .

$$\frac{d}{dx} [Sa \left(\frac{1}{\omega_a} - 1 \right) \frac{R}{M_b} T] = Sa \left(\frac{1}{\omega_a} - 1 \right) g \quad (\text{III.3})$$

$$\frac{d^2 T}{dx^2} - \frac{Ps h}{k Ap} (T - T_\infty) - \frac{h_f g A_c}{k Ap} \frac{d}{dx} \left[\frac{D}{\omega_a(\omega_a - 1)} \frac{d\omega_a}{dx} \right] = 0 \quad (\text{III.8})$$

To solve these two equations simultaneously it was advantageous to expand and differentiate equation III.3 and to expand and rearrange equation III.8 thereby generating a system of two second order differential equations in ω_a and T.

After differentiating equation III.3 a considerable amount of algebra led to the following form:

$$\begin{aligned} \frac{d^2T}{dx^2} \left\{ \left(\frac{1}{\omega_a} - 1 \right) \left(T \frac{dS_a}{dT} + S_a \right) \right\} - \frac{d^2\omega_a}{dx^2} \frac{(T S_a)}{\omega_a^2} \\ = \left(1 - \frac{1}{\omega_a} \right) \left(2 \frac{dS_a}{dT} + T \frac{d^2S_a}{dT^2} \right) \frac{(dT)^2}{dx^2} + \left(\frac{2}{\omega_a^2} \frac{dT}{dx} \frac{d\omega_a}{dx} \right) \left(T \frac{dS_a}{dT} + S_a \right) \\ - 2 \frac{TS_a}{\omega_a^3} \left(\frac{d\omega_a}{dx} \right)^2 + \frac{M_b g}{R} \left\{ \left(\frac{1}{\omega_a} - 1 \right) \left(\frac{dS_a}{dT} \frac{dT}{dx} \right) - \frac{S_a}{\omega_a^2} \frac{d\omega_a}{dx} \right\} \quad (\text{III.9}) \end{aligned}$$

Equation III.8 was manipulated into a similar convenient format as:

$$\begin{aligned} \frac{d^2T}{dx^2} - \frac{d^2\omega_a}{dx^2} \left\{ \frac{Ac h_{fg}}{Ap k (\omega_a - 1) \omega_a} \right\} &= \frac{Ps h}{k Ap} (T - T_\infty) \\ + \frac{Ac h_{fg}}{Ap k (\omega_a - 1) \omega_a} \frac{d\omega_a}{dx} \left\{ -D S_a \left(\frac{d\omega_a}{dx} \right) \left(\frac{1}{\omega_a - 1} + \frac{1}{\omega_a} \right) \right. \\ \left. + (S_a \frac{dD}{dT} + D \frac{dS_a}{dT}) \frac{dT}{dx} \right\} \quad (\text{III.10}) \end{aligned}$$

Equations III.9 and III.10 represent the final form of the desired equations in a dimensional format.

B. BOUNDARY CONDITIONS

To numerically integrate these second order differential equations four boundary conditions must be established at $x = 0$. To avoid the obvious singularity when $\omega_a = 1.0$ (at the evaporator end of the condenser) integration must proceed from the condenser-reservoir end toward the evaporator. Therefore $x = 0$ is chosen to be the condenser-reservoir end of the heat pipe.

Appropriate boundary conditions are then as follows.

$$1. \quad T = T_{\infty} \quad \text{at } x = 0 \quad (\text{III.11})$$

$$2. \quad \frac{dT}{dx} = 0 \quad \text{at } x = 0 \quad (\text{III.12})$$

$$3. \quad \frac{d\omega_a}{dx} = 0 \quad \text{at } x = 0 \quad (\text{III.13})$$

$$4. \quad \omega_a = \omega_{a_0} > 0 \quad \text{at } x = 0 \quad (\text{III.14})$$

The selection of an initial value of ω_a is somewhat arbitrary. However, the initial value of ω_a selected, in conjunction with the values of T_e and T_{∞} used in the model, indirectly fix the amount of noncondensable gas and the operating power level of the heat pipe being simulated for a particular working fluid - noncondensable gas pair.

C. NONDIMENSIONALIZATION OF EQUATIONS

To nondimensionalize equations III.9 and III.10 the following dimensionless variables were defined.

$XS = x/L$ nondimensional length
(independent variable)

$Y1 = \frac{T - T_{\infty}}{T_e - T_{\infty}}$ nondimensional temperature

where:

$$T = T(x)$$

T_e = prescribed evaporator temperature

T_{∞} = ambient temperature

$Y2 = \ln \omega_a$ logarithm of the mass fraction of species 'a'

For ease of notation and computer programming the first and second derivatives of the two dependent variables were represented as follows.

$$Y3 = \frac{d(Y1)}{dXS} = DY1$$

$$Y4 = \frac{d(Y2)}{dXS} = DY2$$

$$DY3 = \frac{d(DY1)}{dXS} = \frac{d^2(Y1)}{dXS^2}$$

$$DY4 = \frac{d(DY2)}{dXS} = \frac{d^2(Y2)}{dXS^2}$$

Each function of temperature was nondimensionalized with respect to the function value at evaporator temperature (T_e).

The following nondimensionalized functions of temperature were then developed for direct substitution into equations III.9 and III.10.

$$G_1 = \frac{D}{D(T_e)} \quad \text{nondimensional binary mass diffusion coefficient}$$

$$G_2 = \frac{d(D)}{dT} \frac{(T_e - T_\infty)}{D(T_e)} \quad \text{nondimensional temperature gradient of the binary mass diffusion coefficient}$$

$$G_3 = \frac{h_{fg}}{h_{fg}(T_e)} \quad \text{nondimensional heat of vaporization}$$

$$G_4 = \frac{S_a}{S_a(T_e)} \quad \text{nondimensional working fluid density}$$

$$G_5 = \frac{d(S_a)}{dT} \frac{(T_e - T_\infty)}{S_a(T_e)} \quad \text{nondimensional temperature gradient of working fluid density}$$

$$G_6 = \frac{d^2(S_a)}{dT^2} \frac{(T_e - T_\infty)^2}{S_a(T_e)} \quad \text{nondimensional curvature of working fluid saturation density curve}$$

$$G_7 = \frac{h}{h(T_e)} \quad \text{nondimensional convection heat transfer coefficient}$$

Using the nondimensional variables and functions of temperature defined above, equations III.9 and III.10 were transformed into the following nondimensionalized system of two equations in two dimensionless dependent variables (Y_1 and Y_2), and their first and second derivatives with respect to axial (dimensionless) length.

$$\begin{aligned}
 & DY3 \{ (1 - \text{Exp}(Y2)) ((K1Y1+1)G5 + K1G4) \} - DY4((K1Y1+1)G4) \\
 & = (1-\text{Exp}(Y2)) \{ Y3(Y3(2K1G5+(K1Y1+1)G6)+K2G5) \} \\
 & + Y4 \{ 2Y3((K1Y1+1)G5+K1G4) - G4((K1Y1+1)Y4-K2) \} \quad (\text{III.15})
 \end{aligned}$$

$$\begin{aligned}
 & DY3(\text{Exp}(Y2)-1) - DY4(K3G1G3G4) = (\text{Exp}(Y2)-1)K4G7Y1 \\
 & + G1G3Y4(K3(G4(\frac{G2Y3}{G1} - \frac{Y4 \text{Exp}(Y2)}{(\text{Exp}(Y2)-1)}) + G5Y3)) \quad (\text{III.16})
 \end{aligned}$$

where K_1 through K_4 are dimensionless constants:

$$\begin{aligned}
 K_1 &= \left(\frac{T_e - T_\infty}{T_\infty} \right) \\
 K_2 &= \left(\frac{M_b g L}{R T_\infty g \zeta} \right) \\
 K_3 &= \left(\frac{A_c h_{fg}(T_e) D(T_e) S_a(T_e)}{k A_p (T_e - T_\infty)} \right) \\
 K_4 &= \left(\frac{P_s h(T_e) L^2}{k A_p} \right) \quad (\text{III.17})
 \end{aligned}$$

In terms of the nondimensional variables, the boundary conditions (equations III.11 through III.14) became:

$$1. \quad Y_1 = 0 \quad \text{at } X_S = 0 \quad (\text{III.18})$$

$$2. \quad Y_2 = Y_{2_0} < 0 \quad \text{at } X_S = 0 \quad (\text{III.19})$$

$$3. \quad Y_3 = D Y_1 = 0 \quad \text{at } X_S = 0 \quad (\text{III.20})$$

$$4. \quad Y_4 = D Y_2 = 0 \quad \text{at } X_S = 0 \quad (\text{III.21})$$

where $Y_2 = \ln \omega_a$. As noted previously with the dimensional form of this boundary condition (equation III.14), the initial value of ω_a selected is arbitrary. However the initial value of ω_a , in conjunction with the specified values of T_e and T_∞ , indirectly specifies the quantity of noncondensable gas and operating power level of the heat pipe. Since $0 < \omega_a < 1.0$, the corresponding value of Y_2 must be negative.

IV. DISCUSSION AND RECOMMENDATIONS

A. DISCUSSION

The Continuous System Modeling Program (CSMP), developed by the IBM Company [5], is a straightforward computer package available to numerically solve equations III.15 and III.16. The CSMP package includes a number of optional numerical integration methods including a variable step fourth-order Runge-Kutta method.

Although equations III.15 and III.16 are highly nonlinear in the dependent variables Y1 and Y2, this system of equations can be considered linear in the second derivatives of Y1 and Y2. Therefore these equations can be written in the following form:

$$(COEF1)(DY3) + (COEF2)(DY4) = F1 \quad (IV.1)$$

$$(COEF3)(DY3) + (COEF4)(DY4) = F2 \quad (IV.2)$$

where,

$$COEF1 = \{(1-Exp(Y2))((K1Y1+1)G5+K1G4)\} \quad (IV.3)$$

$$COEF2 = -((K1Y1+1)G4) \quad (IV.4)$$

$$\begin{aligned}
 F1 &= (1-\text{Exp}(Y2)) \{ Y3(Y3(2K1G5+(K1Y1+1)G6)+K2G5) \} \\
 &\quad + Y4\{2Y3((K1Y1+1)G5+K1G4)-G4((K1Y1+1)Y4 - K2)\}
 \end{aligned} \tag{IV.5}$$

$$COEF3 = (\text{Exp}(Y2)-1) \tag{IV.6}$$

$$COEF4 = -(K3G1G3G4) \tag{IV.7}$$

$$F2 = (\text{Exp}(Y2)-1)K4 G7 Y1 + G1 G3 Y4 (K3(G4$$

$$\cdot \left(\frac{G2 Y3}{G1} - \frac{Y4 \text{Exp}(Y2)}{(\text{Exp}(Y2)-1)} + G5 Y3 \right) \tag{IV.8}$$

Using these definitions, the nondimensionalized equations representing steady state heat pipe operation may be numerically integrated using the variable step fourth-order Runge-Kutta option of the CSMP routine as follows.

$$Y1 = \int_0^1 (DY1) dXS = \int_0^1 \frac{d(Y1)}{dXS} dXS \tag{IV.9}$$

$$Y2 = \int_0^1 (DY2) dXS = \int_0^1 \frac{d(Y2)}{dXS} dXS \tag{IV.10}$$

$$Y3 = \int_0^1 (DY3) dXS = \int_0^1 \frac{d^2(Y1)}{dXS^2} dXS \tag{IV.11}$$

$$Y_4 = \int_0^1 (DY_4) dx_S = \int_0^1 \frac{d^2(Y_2)}{dx_S^2} dx_S \quad (IV.12)$$

where,

$$DY_1 = Y_3 \quad (IV.13)$$

$$DY_2 = Y_4 \quad (IV.14)$$

and by Cramer's rule,

$$DY_3 = \frac{(F_1 COEF_4 - F_2 COEF_2)}{(COEF_1 COEF_4 - COEF_2 COEF_3)} \quad (IV.15)$$

$$DY_4 = \frac{(F_2 COEF_1 - F_1 COEF_3)}{(COEF_1 COEF_4 - COEF_2 COEF_3)} \quad (IV.16)$$

This model can be used to demonstrate gravitational effects on the vapor-gas interface region by providing axial profiles of temperature and the mass concentration of the working fluid. When the molecular weight of the noncondensable gas is less than that of the working fluid, the axial profiles will show a fairly small interface region between vapor and gas. However, when the molecular weight of the noncondensable gas is substantially greater than that of the working fluid, the profiles will show a distorted (spread out) interface region.

Qualitative results can be obtained from this model by using additional simplifying assumptions and built-in CSMP functions as follows:

1. Nominal constant values can be selected to represent the convection heat transfer coefficient and the binary mass diffusion coefficient. For example,

$$h(T) = 1.0 \text{ BTU/HR/FT}^2 \text{ for free convection}$$

$$D(T) = 0.4 \text{ FT}^2/\text{HR for Krypton-Methanol}$$

or

$$D(T) = 1.9 \text{ FT}^2/\text{HR for Helium-Methanol}$$

2. To generate values of working fluid density (S_a) and heat of vaporization (h_{fg}) at any temperature, a CSMP function generator can be used to provide quadratic interpolation of tabulated saturation properties. Tabulated thermodynamic properties of methanol are available in reference 7. The CSMP function "SLOPE" can be used to generate values of $d(S_a)/dT$ and $d^2(S_a)/dT^2$ as functions of temperature.

The temperature profile generated by this model for a given vertical heat pipe using a particular working fluid-noncondensable gas pair, with given input values of T_e , T_∞ , and $\omega_a(0)$, represents heat pipe operation at a certain power level and total pressure with a specific load of non-condensable gas. The power level, total pressure, and mass of noncondensable gas represented could be computed as follows.

$$\text{Mass of Gas (lb}_m\text{)} = \frac{\pi}{4} D_v^2 L \int_0^1 \frac{s_a}{\omega_a} (1-\omega_a) dx_s \quad (\text{IV.17})$$

$$\text{Power Level (BTU/HR)} = P_v L \int_0^1 \dot{m}_a h_{fg} dx_s \quad (\text{IV.18})$$

$$\text{Total Pressure (psia)} = F_a(T_\infty) + \frac{s_a(T_\infty)}{\omega_a(0)} (1-\omega_a(0)) \frac{R T_\infty}{M_b} \quad (\text{IV.19})$$

B. RECOMMENDATIONS

Based upon preliminary work using CSMP, numerical results from this model could be improved by the following measures.

1. Analytical expressions are recommended in lieu of the special CSMP functions to represent working fluid saturation properties as functions of temperature. Expressions for the first and second derivatives of density as functions of temperature could then be found analytically and used directly. The resulting expressions should yield more accurate values, particularly for the second derivative of density, than are achievable through multiple use of the CSMP "SLOPE" function with tabulated saturation values.
2. Since the binary mass diffusion coefficient is a fairly strong function of temperature, quantitative results in the interface region could be improved considerably by use of an empirical expression for $D(T)$ instead of a nominal constant value. Reference 8 contains appropriate empirical expressions for binary mass diffusion coefficients.

3. Quantitative results could also be improved somewhat by using an appropriate empirical expression for the convection heat transfer coefficient as a function of temperature instead of a nominal constant value. Reference 9 provides a variety of expressions for $h(T)$ depending upon the convective condition being simulated.

4. With the improvements noted above, this model could be used in an iterative loop to obtain preliminary quantitative design information concerning vertically oriented terrestrial heat pipes when the molecular weights of the operating substances are substantially different.

For example, if the working fluid-noncondensable gas pair is specified, along with the required heat source (evaporator) temperature range, ambient conditions, and the maximum safe total pressure for structural integrity of the pipe, then equation IV.19 could be used to determine the value of $w_a(0)$ to use for boundary condition III.19. Equation IV.17 could then be used to numerically compute the required mass of non-condensable gas to load depending upon the heat pipe dimensions. Equation IV.18 would then yield the maximum heat rejection rate obtainable for these conditions. If this heat rejection rate is not adequate for the design requirements, the designer could vary any of a number of parameters, such as operating temperature, pipe dimensions, total pressure allowed, substances used, etc., until an adequate design is found by iteration.

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